A Cost-Criticality Based (Max,+) Optimization Model for Operations Scheduling

Karla Quintero, Eric Niel, José Aguilar, and Laurent Piétrac

Abstract The following work proposes a (max,+) optimization model for scheduling batch transfer operations in a flow network by integrating a cost/criticality criterion to prioritize conflicting operations in terms of resource allocation. The case study is a seaport for oil export where real industrial data has been gathered. The work is extendable to flow networks in general and aims at proposing a general, intuitive algebraic modeling framework through which flow transfer operations can be scheduled based on a criterion that integrates the potential costs due to late client service and critical device reliability in order to satisfy a given set of requests through a set of disjoint alignments in a pipeline network. The research exploits results from previous work and it is suitable for systems handling different client priorities and in which device reliability has an important short-term impact on operations.

Key words: algebraic modeling, flow networks, oil pipeline networks, (max,+) theory, schedule optimization, system reliability.

1 Introduction

The following work continues the developments in [18] and [19] in which a (max, +) optimization model for scheduling transfer operations on a flow network was

Karla Quintero
THALES GROUP, Vélizy, 78141 France, e-mail: karla.quintero@thalesgroup.com

Eric Niel
INSA, Lyon, 69621, France, e-mail: eric.niel@insa-lyon.fr

José Aguilar
ULA, Mérida, 5101, Venezuela, e-mail: aguilar@ula.ve

Laurent Piétrac
INSA, Lyon, 69621, e-mail: laurent.pietrac@insa-lyon.fr

proposed. The case study continues to be a seaport for oil export in which oil batches must be transported between 2 different points through a path in an intricate pipeline network.

Given that different oil batches must not mix, conflict in resource allocation arises in order to process more requests than the network is able to handle simultaneously. Some approaches to manage resource allocation conflicts are Petri Nets, specifically event graphs, where conflicts are previously solved through a routing policy. This routing policies are criteria allowing to choose a transition among a group of conflicting ones demanding to be fired. Naturally, the routing policy must be coherent with the special needs of each system to be modeled; see [2], [14], and [1] for an overview on common routing policies. Some heuristic approaches can also be considered; for instance, [16] implements an ant colony optimization algorithm in which conflict is modeled as a probabilistic choice rule depending on the pheromone trail and a heuristic function.

In [18] and [19], conflict resolution has been intuitively modeled for a flow network through (max,+) algebra. In these developments, industrial data indicates that in case of delayed service the seaport incurs into a different monetary penalty depending on the client. In those results, the objective was to find a schedule minimizing the Total Cost due to Penalties (TCP) incurred by the seaport. In [18], fixed preventive maintenance tasks on valves were considered as constraints in the model in order to schedule oil transfer operations optimally. In [19], maintenance relaxation was explored in order to obtain better global schedules through a trade-off between satisfying maintenance operations and satisfying a given set of clients implying some potential costs in the case of delays. Also, in that work, conflict resolution in resource allocation was explored based on the Total Potential Penalty (TPP) of each client, i.e. a product (in conventional algebra) between the request's processing time and the penalty per time unit.

In this paper, we explore the integration of failure risk into the already established penalty-based framework. More specifically, here an analogy is done with an approach, proposed in [13], used to prioritize maintenance tasks on devices and it is modified in order to prioritize conflicting oil transfer operations. Namely, operations are proposed to be prioritized according to an index reflecting the failure probability of the underlying alignment in the network by the monetary consequence which is associated to potential penalties. In order to do so, the failure probability of an alignment must be established, for which we rely on some previous work on alignment search techniques for the case study.

Some approaches, other than (max,+) based, formulating similar optimization problems are: [21], where an optimization model for flow-shop scheduling with setup times is formulated as sets of recursive constraints expressing the dependency between completion times for jobs on machines, and [22] and [23], with classic resource conflict constraints where decision variables impose a precedence between machine operations. The fundamental ideas of these approaches are similar to the proposed model but with the algebraic structure provided by the (max,+) approach constraint formulations can be intuitively built and additional and more intricate phenomena can be easily integrated.

To our knowledge, no similar work has been developed for this type of system, other than the foundations in [18] and [19] which constitute the base of this work. The results are extendable to applications to flow networks of different nature. The developments in this work are part of a larger research scope aiming at optimizing operations in a more complex framework with industrial application in the oil sector.

Firstly, we present the case study in Sect. 2. Section 3 shows some basic notions on (max,+) algebra. Section 4 presents the proposed (max, +) optimization model and the proposed criterion for prioritizing conflicting operations based on penalty costs and alignment failure probability, which is presented in Sect. 5. Results are shown in Sect. 6 and Sect. 7 presents concluding remarks.

2 Case Study

The case study is a seaport for oil export, but the work is be extendable to flow networks of different nature. Oil batches requested by clients must be transported from a set of tanks to a set of loading arms placed at the docks of the seaport through an intricate pipeline network. It is considered that oil flows by gravity through the pipeline network as it is the case of some real seaports.

2.1 Oil Transfer Aspects

An oil transfer operation represents the transfer of a requested oil batch (of a specific type and quantity) from a tank to a specific dock. In reality, a dock may be equipped with one or several loading arms which load the oil batch into the tanker that requests it. Here, it is considered only one loading arm per dock. Each tanker has a loading deadline to be respected which, if exceeded, implies a monetary penalty incurred by the seaport. This penalty is related to the time delay and also to the client's priority. Each of these requests is fulfilled through the selection of an alignment (i.e. a path) in the oil pipeline network, which implies opening the valves included in this alignment and closing all adjacent valves, in order to isolate it from the rest of the network since two types of oil must not mix[1]. From industrial data it is known that oil transfer operations take hours, whereas valve commutations are assumed to take seconds. In this work, it is considered that the alignment is previously established for each transfer operation.

Considerable effort has been devoted to optimizing other features for the case study, most of the results being adaptable for flow networks in general. [20] can be consulted for generic alignment selection maximizing operative capacity (i.e.

[1] a specific scenario is the mixture of two identical oil types. However, oil mixture is not allowed in any scenario since sharing an alignment's section by two transfer operations could result in lower product flow rate and several aspects such as pumping power and pipeline dimensions would have to be considered and are not the focus of this work.

simultaneous disjoint alignments) in the network and [17] for generic alignment selection maximizing operative capacity while minimizing failure risk on valves. For illustration purposes on the system configuration, Fig. 1(a) shows an example of a simplified oil seaport and Fig. 1(b) shows the network model as an undirected graph where arcs represent the valves and the nodes represent the linked pipeline segments.

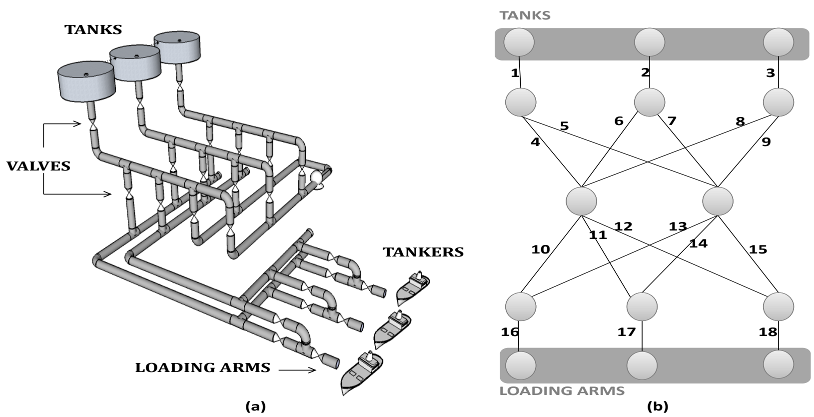

Fig. 1 Oil seaport example

2.2 *Conflicts in Resource Allocation*

Simultaneous alignments for two or more requests must be disjoint since different oil batches must not mix. The work in [18] yields the following definition.

Definition 1. Two or more alignments (for oil transfer) are in conflict if they share at least one valve and if either the valve requires different states for different alignments or if it requires being open for more than one alignment.

Fig. 2(a) (from [18]) shows two disjoint alignments to satisfy requests R_1 and R_3. Solid lines illustrate the valves to open and dotted lines (of the same tone) the valves to close in order to isolate the alignment; e.g.: to enable the alignment for R_1 valves 1, 4, 10, and 16 must open and valves 5, 6, 8, 12, 11, and 13 must close. In Fig. 2(a), no conflict arises for any valve since the common resources (valves 5, 8, 12, and 13) are all valves to be closed, therefore they can enable both transfer operations simultaneously.

On Fig. 2(b), another request (R_2) is added and conflicts arise for valves 10 and 16, since they should open for 2 transfer operations (therefore, mixing 2 oil types), and for valves 4 and 6, since the required commutations are different for both trans-

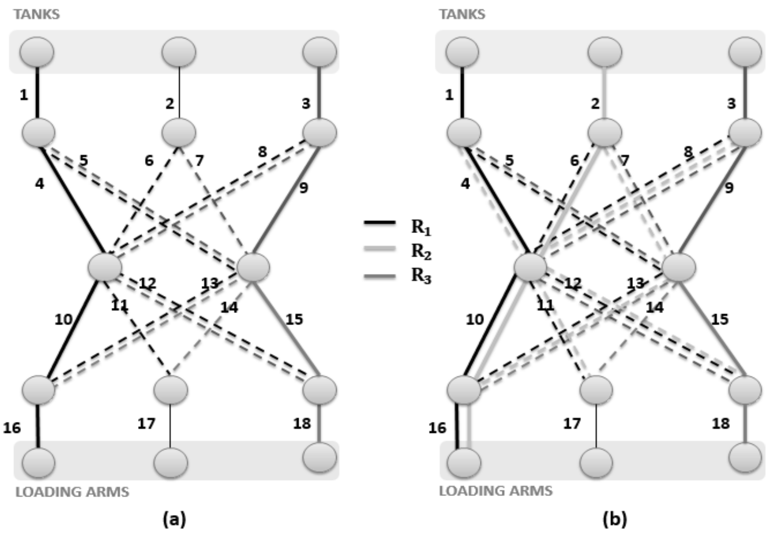

Fig. 2 Conflict representation in a undirected graph

fer operations (which is physically impossible); therefore, R_1 and R_2 cannot be processed simultaneously.

2.3 Scheduling Oil Transfer Operations on a Seaport - Penalty-Related Aspects

In this work, resources of interest are valves and their availability is determined by their allocation by different alignments aiming at satisfying oil transfer operations for several clients. Client requirements include deadlines for tanker loading, which in case of violation by the seaport imply monetary penalties.

For each client, a negotiation occurs with the seaport. In this phase, the client imposes (within certain conditions not relevant to this work) for a specific tanker, the penalty to be paid by the seaport in case of delay (in thousands of dollars per hour) caused by the seaport. At the same time, the seaport imposes a time window of three days within which the tanker can arrive and be immediately docked and served. From the moment of arrival within this time window, the maximum service time for every tanker is 36 hours for loading and 4 hours for paperwork. Since the focus of this paper is on seaport transfer operations, the paperwork interval is discarded and the focus is on the maximum loading interval of 36 hours as the deadline for each tanker. From that point on, every extra hour invested in the service of the tanker will result in a penalty for the seaport, if the delay has been indeed caused by the seaport. Conversely, if the service of a tanker surpasses the 36 hours due to tanker's technical difficulties, then the client pays the seaport a penalty for dock over-occupation. Operations management on the seaport contributes to the general objective of profit maximization but client-incurred-penalties do not represent in any way an optimization objective, i.e. they are unexpected events which the seaport does not aim at maximizing through operations' scheduling. If the tanker arrives after its time window, the seaport does not incur into any penalties for the waiting time for the tanker to be served. No further information has been granted concerning other arrival scenarios and possible consequences in the service.

3 Preliminaries on (max,+) Algebra

This section provides a (max, +) theory overview allowing to understand the basis of this mathematical modeling technique with application to the scheduling problem approached in the research.

(max, +) algebra is defined as a mathematical structure denoted as R_{max}, constituted by the set $R\bigcup\{-\infty\}$ and two binary operations $\oplus$ and $\otimes$, which correspond to maximization and addition, respectively. This algebraic structure is called an idempotent commutative semifield. As [3] states, a semifield $\mathcal{K}$ is a set endowed with two generic operations $\oplus$ and $\otimes$ complying with certain classic algebraic properties. The zero element is $\varepsilon = -\infty$, and the identity element is $e = 0$. The main properties of this algebraic structure (similar to the ones defined in conventional algebra) are:

Operation $\oplus$:

- is associative (e.g. $a \oplus (b \oplus c) = (a \oplus b) \oplus c$),
- is commutative (e.g. $a \oplus b = b \oplus a$),
- has a zero element ε (e.g. $a \oplus \varepsilon = a$),
- is idempotent (i.e. $a \oplus a = a$; $\forall a \in \mathcal{K}$).

Operation $\otimes$:

- is distributive with respect to $\oplus$ (e.g. $a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c)$),
- is invertible. For example, in (max,+) algebra: if $2 \otimes 3 = 5$ then $2 = 5 \oslash 3$ or in conventional notation: if $2 + 3 = 5$ then $2 = 5 - 3$ (here, operator $\oslash$ denotes the inverse of the $\oplus$ operation),
- has an identity element e which satisfies $\varepsilon \otimes e = e \otimes \varepsilon = \varepsilon$.

Some equivalent (max,+) and conventional algebra expressions are the following:

$$\begin{array}{cc}
a \oplus b \Leftrightarrow max(a,b) & a \otimes b \Leftrightarrow a + b \\
a \oplus \varepsilon \Leftrightarrow max(a,\varepsilon) = a & a \otimes \varepsilon \Leftrightarrow a + \varepsilon = \varepsilon \\
a \oplus e \Leftrightarrow max(a,e) = a & a \otimes e \Leftrightarrow a + e = a
\end{array}$$

Based on the aforementioned basic operations, more intricate ones are defined in the context of the algebraic structure such as matrix product for example. Synchronization phenomena can be modeled in a very straightforward fashion through (max,+) algebra which has led to a very wide application to transportation systems. However, research in this field continues to explore further possibilities.

For the purposes of this research, the interest is on the application of the modeling technique to a system in which resource allocation conflicts constitute the main feature.

The application of this theory to discrete event systems exhibiting synchronization phenomena leads to the formulation of very intuitive (max,+)-linear models formed by equations such as $x_3 = x_1 \otimes \tau_1 \oplus x_2 \otimes \tau_2$. In this equation, x_i is the start date of an event i, and τ_i is its duration. x_i is usually denoted as *'dater'* in the (max, +) context. In this example, the dater of event 3 is given by the maximum of the completion times of events 1 and 2; which can be interpreted as the synchronization of 2 tasks or 2 task sequences (e.g. a train that only departs when 2 other trains arrive at the station with connecting passengers).

With the principle shown in the former equation, a (max,+)-linear system model describing the interactions among all relevant tasks or processes can be obtained in the form of $X = AX$, where X is the variables vector (i.e. $X = [x_1\ x_2 \ldots\ x_n]^T$) and A is the matrix containing all time relations between the variables. Analogies with classic linear system theory would be applicable to this simple model by considering maximization and addition as basic operations, as well as all the aforementioned properties in the algebraic structure.

(Max,+) theory is a research field that has caught the attention of the scientific community for its intuitive modeling potential of discrete event systems' phenomena that would usually involve more intricate mathematical models. For further information on (max, +) algebra for production chains and transportation networks [4] can be consulted. [3] can be consulted for (max, +)-linear system theory, [7] for (max, +) theory applied to traffic control, [15] for an application to production scheduling in manufacturing systems, and [10] for maintenance modeling for a helicopter. Moreover, considerable effort has been dedicated to exploiting the potential of (max, +) algebra combined with automata theory, leading to the study of (max, +) automata which can also be applicable to schedule optimization problems; see [5], [11], and [8] for developments in this field.

4 (max,+) Optimization Model

The basis for the mathematical optimization model used in this paper have been defined in [18]. In this work, maintenance aspects are not considered and the main objective is to prioritize operations according to a relation between the potential penalty due to late service and the reliability of the network alignment. In [18] the purpose was to minimize the TCP (Total Cost due to Penalties), which was defined as the total cost in which the seaport incurs due to late service of a set of clients for a time horizon. One of the main set of constraints proposed is the one modeling conflicts in resource allocation, presented in (1) in conventional algebra and in (2) in (max,+) algebra [2], where $\mathfrak{I}$ is the set of all possible requests.

$$x_i = max\Big(t_0\ ; u_i\ ; max_{i'}\left(x_{i'} + p_{i'} + zp_{i'} + zc_{i'} + V_{i,i'}\right)\Big)\ \ \forall\ i, i' \in \mathfrak{I} |\ i \neq i' \tag{1}$$

$$x_i = t_0 \oplus u_i \oplus\ \left(\bigoplus_{i'}\left(x_{i'} \otimes p_{i'} \otimes zp_{i'} \otimes zc_{i'} \otimes V_{i,i'}\right)\right)\ \ \forall\ i, i' \in \mathfrak{I} |\ i \neq i' \tag{2}$$

The aforementioned equivalent constraints determine the start date (x_i), also called *'dater'* in the (max, +) context, to satisfy a request i. Variables include:

- x_i: dater for an oil transfer operation, also called request i,
- $x_{i'}$: dater for a conflicting transfer operation i' requesting common resources to operation i,
- $V_{i,i'}$: decision variable that defines the precedence between two conflicting oil transfer operations i and i',
- u_i: arrival date for the tanker for request i,
- $zp_{i'}$, and $zc_{i'}$ represent, respectively, the possible unexpected delays in client service due to technical difficulties in the seaport and due to technical difficulties within the tanker.
- Parameters include: t_0, and $p_{i'}$ which respectively correspond to the start date of the scheduling time horizon, and the nominal duration of the oil transfer operation.

Equation (2) states that the start date for an oil transfer operation will depend on the start date of the time horizon for scheduling, the arrival date of the tanker in the seaport, and the completion time of all conflicting oil transfer operations which are to be executed before request i. Notice that for all conflicting operations interruption variables model the possible delays that could arise in the execution of operations. All decision variables are binary, taking the values $e = 0$ or $\varepsilon = -\infty$, as the identity and zero elements defined in (max,+) theory. For instantiation purposes, values are *zero* or B, so that B is a very large negative real number.

Moreover, each decision variable has a complementary one (e.g. if $V_{i,i'} = e$, then $V_{i',i} = B$ or vice versa). For example, in (2), when $V_{i,i'} = B$ the entire third term of the global maximization is negligible, which implies that the completion time of operation $x_{i'}$ is not relevant to calculate x_i, indicating that request i will be executed before request i'. This value assignment would automatically generate the value assignment of the complementary decision variable (i.e. $V_{i',i} = e$) which means that in the constraint to determine $x_{i'}$ the completion time of operation i would indeed be taken into account.

$$V_{i,i'} \otimes V_{i',i} = B \ \ \forall\ i, i' \in \mathfrak{I} \tag{3}$$

$$V_{i,i'} \oplus V_{i',i} = e \ \ \forall\ i, i' \in \mathfrak{I} \tag{4}$$

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[2] taking into consideration that maintenance is not approached in this work

Equations (3) and (4) restrict the values of the decision variables to be either *zero* or B for potential conflicts between two transfer operations.

$$D_i = \begin{cases} u_i \otimes 36 & \forall\ i \in \mathfrak{I} | u_i \in tw_i \\ x_i \otimes 36 & \forall\ i \in \mathfrak{I} | u_i > utw_i \\ ltw_i \otimes 36 & \forall\ i \in \mathfrak{I} | u_i < ltw_i \end{cases} \tag{5}$$

$$dpr_i = (x_i \otimes p_i \otimes zp_i \otimes zc_i \oslash D_i) \oplus e \qquad \forall i \in \mathfrak{I} \tag{6}$$

In (5) the deadline D_i for a request i is modeled where $tw_i = [ltw_i, utw_i]$ is the authorized time window of three days for the tanker's arrival. Since no further information has been gathered on deadlines given early arrival of the tanker, it has been considered that the 36 hours for loading start at the beginning of the authorized time window.

The *delay per request* (dpr) is determined in (6) which is the difference between a request's completion time (including the possible delays caused by the seaport and/or the client) and its deadline. In [18], Hypothesis 1 was proposed to deal with combined delays between the seaport and the client. Within this context, (7) modeled the *penalized delay for the seaport* (pds) per request; i.e. the time interval (hours) for which the seaport will actually incur into penalties.

Hypothesis 1. the dock over-occupation penalty per hour per client (paid by each client) is considered equal to the penalty per hour for that same client paid by the seaport in the case of delay caused by the seaport.

$$pds_i = \begin{cases} \oslash \left[\left(\oslash zu_i \oslash zp_i \otimes zc_i \right) \oplus \left(\oslash dpr_i \right) \right] & \forall (zu_i \otimes zp_i) > zc_i \\ e & otherwise \end{cases} \tag{7}$$

Notice that (5-7) allow to determine the TCP or *Total Cost due to Penalties* as presented in (8), which has already proven to be a crucial metric in operations management. Equation (8) is the (max, +) algebra representation for the sum of the products of each penalized delay (in hours) and its corresponding penalty (in \$/hour).

$$Min\ TCP = \bigotimes_i \left(\bigotimes_{n=1}^{pds_i} c_i \right) \quad \forall i \in \mathfrak{I} \tag{8}$$

In [18], the optimal schedule for oil transfer operations was obtained while considering a fixed preventive maintenance program to be respected. In [19], some (max,+)-linear representations of the model were obtained through value assignment of decision variables based on a routing (or conflict resolution) policy which consisted on prioritizing operations with the greatest TPP (Total Potential Penalty), defined as the product of the nominal duration and the penalty per time unit for the tanker. In this work, the focus lies on exploring a routing policy that integrates reliability data of the network section of interest with related potential costs. Namely, failure probability on each alignment is considered in order to estimate consequent costs (measured as potential penalties for late service due to failure of the predefined alignment) and hence prioritize operations according to a failure/cost relation.

The basic premise is that this approach is useful in systems where device reliability is a very influential metric when it comes to managing operations. This could be related with 'forced production' situations, in which maintenance is forced to be delayed due to operational requirements and therefore device reliability is crucial in carrying out operations in the network. Given such scenario, in which device conditions are susceptible to influence operational performance, alignment failure consequently implies potential penalties for the request that is being processed with such alignment. This approach is the result of an analogy applied with a similar approach used to prioritize maintenance activities on devices as it is stated in [13]. In the developments therein, an index called CBC (Cost-Based Criticality) is used to rank maintenance tasks based on the device's failure probability and its consequence. The index is computed as the product between the device's failure probability and the consequent monetary costs that arise due to failure (in which production losses, environmental impact, quality loss, are considered among other costs).

In this work, a similar index is used as a routing policy to solve conflicts between oil transfer operations with alignments sharing common resources. Alignments are considered to be previously defined for each request. Alignment's failure probability is computed based on previous work on alignment search for the case study and the monetary consequence of failure is considered as the TPP (Total Potential Penalty) for each specific client. The aforementioned index is denoted in this work as PCI (Penalty-Criticality Index) and is defined in (9), where $TPP_i = p_i \times c_i$ (as defined in [19]) which is the product between the processing time for operation i and the penalty per hour for such client, and $P_{f(i)}$ is the failure probability of the alignment assigned to process such request.

$$PCI_i = TPP_i \times P_{f(i)} \tag{9}$$

5 Failure Probability for an Alignment

In [17], an approach was proposed to find the greatest set of independent simultaneous alignments (also called maximum operative capacity) in a pipeline network while minimizing failure risk for the same case study. The approach was based on a minimum flow cost algorithm in which costs were related to devices' reliability and flow was considered to be either existent or nonexistent on pipeline segments (i.e. no flow rate was managed).

For an alignment to function properly in order to carry out an oil transfer operation, all valves in the alignment should be able to commute to the 'open' state properly and all adjacent valves should commute to the 'closed' state properly, in order to isolate the alignment. Considering that proper commutation behavior on each valve is independent from all others and that it can be described as a random variable, an alignment's estimation of a well-functioning probability can be described as the product of the well-functioning probabilities for each and every one of the valves involved. This metric is defined in (10), where $v \in A$ stands for all valves involved in alignment A.

For example, in the case of Fig. 2, for the alignment for request R_1, the well-functioning probability ($P_{w(i)}$) would be $P_{w(i)} = P_1 \times P_4 \times P_{10} \times P_{16} \times P_5 \times P_6 \times P_8 \times P_{12} \times P_{11} \times P_{13}$. Consequently, the alignment's failure probability is defined as $P_{f(i)} = 1 - P_{w(i)}$. It is fundamental to understand that this metric is used exclusively for differentiation purposes among alignments and their condition in order to execute a set of given requests.

$$P_{w(i)} = \prod_{(v \in A)} P_{f(v)} \tag{10}$$

This approach is in no way restrictive, and the well-functioning probability could be determined otherwise for a different system, and could be fed with the proper probability estimations in each flow network according to condition monitoring results on devices in the best case scenario. Moreover, a different criticality level according to the needs of each particular system could be considered in order to prioritize transfer operations.

6 Results

Fig. 3 shows an instance with 7 requests to be executed through the depicted alignments (only open valves are depicted for better comprehension). In this figure, 3 zones are identified as A, B, and C in order to define 3 different probability values for well-functioning behavior on valves (for illustration purposes). Most valves clearly fall into a specific zone, and those that do not are considered as follows: valves 5 and 13 $\in$ Zone A, and 8 and 12 $\in$ Zone C. In Fig. 3, conflicts among alignments can be easily identified, and according to the structure proposed in (2), the set of conflict constraints is obtained in (11-17).

$$x_1 = u_1 \oplus x_2 p_2 V_{1,2} \oplus x_6 p_6 V_{1,6} \oplus x_7 p_7 V_{1,7} \tag{11}$$

$$x_2 = u_2 \oplus x_1 p_1 V_{2,1} \oplus x_5 p_5 V_{2,5} \oplus x_6 p_6 V_{2,6} \oplus x_7 p_7 V_{2,7} \tag{12}$$

$$x_3 = u_3 \oplus x_4 p_4 V_{3,4} \oplus x_5 p_5 V_{3,5} \oplus x_6 p_6 V_{3,6} \oplus x_7 p_7 V_{3,7} \tag{13}$$

$$x_4 = u_4 \oplus x_3 p_3 V_{4,3} \oplus x_5 p_5 V_{4,5} \oplus x_7 p_7 V_{4,7} \tag{14}$$

$$x_5 = u_5 \oplus x_2 p_2 V_{5,2} \oplus x_3 p_3 V_{5,3} \oplus x_4 p_4 V_{5,4} \oplus x_6 p_6 V_{5,6} \tag{15}$$

$$x_6 = u_6 \oplus x_1 p_1 V_{6,1} \oplus x_2 p_2 V_{6,2} \oplus x_3 p_3 V_{6,3} \oplus x_5 p_5 V_{6,5} \oplus x_7 p_7 V_{6,7} \tag{16}$$

$$x_7 = u_7 \oplus x_1 p_1 V_{7,1} \oplus x_2 p_2 V_{7,2} \oplus x_3 p_3 V_{7,3} \oplus x_4 p_4 V_{7,4} \oplus x_6 p_6 V_{7,6} \tag{17}$$

Table 1, presents input data in columns 2, 3 and 4 as it is known in real operational conditions (i.e. the operation's processing time p_i, as well as the penalty per time unit for that specific tanker c_i, and finally the obtained TPP). The failure probability

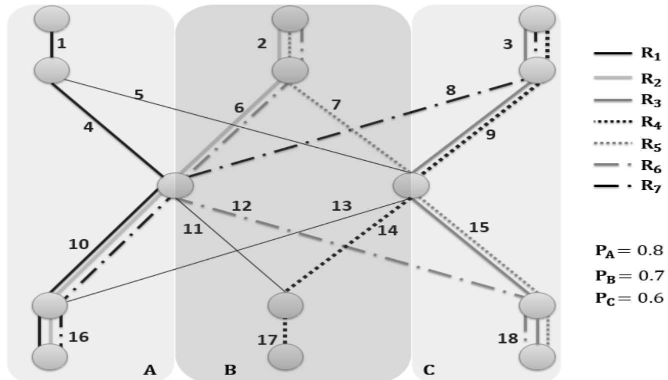

Fig. 3 Alignments for oil transfer operations

Table 1 Input data for linear model validation

Request	p_i (hours)	c_i ($/hour)	TPP_i	$P_{f(i)}$
R_1	20	3000	60000	0.954
R_2	20	3000	60000	0.975
R_3	20	3000	60000	0.985
R_4	20	2000	40000	0.980
R_5	20	2000	40000	0.980
R_6	20	4000	80000	0.980
R_7	20	4000	80000	0.974

of each alignment is computed with the reliability probabilities proposed in Fig. 3 for each valve depending on the zone, and through the approach proposed in Sect. 5, yielding the results on column 5 in Table 1.

In this table, operations' processing times are assumed to be equal to allow a better manual comprehension of the prioritization of conflicting tasks. This is in no way restrictive and it is only assumed for result illustration purposes. The result from the proposed data is the vector of *PCI* indices as $PCI = [57240, 58500, 59100, 39200, 39200, 78400, 77920]^T$.

Assuming the worst case scenario, in which all tankers for all requests arrive at the same time (which is unlikely but holds for illustration purposes), and all within their authorized time windows, then all potential conflicts (due to resource sharing) become actual conflicts that must be dealt with by assignment of decision variable values according to the proposed *PCI* criterion.

Given the obtained *PCI* vector, R_6 is the most pressing operation. Consequently, this operation does not depend on the completion time of other conflicting operations and therefore decision variable assignment must be such that for a conflicting operation i, $V_{6,i} = \varepsilon$.

In (16), this translates into the value assignments: $V_{6,1} = V_{6,2} = V_{6,3} = V_{6,5} = V_{6,7} = \varepsilon$, which automatically yields the assignments of all complementary variables in all other equations, i.e. $V_{1,6} = V_{2,6} = V_{3,6} = V_{5,6} = V_{7,6} = e$. Analogously, all remaining decision variables values are assigned according to the *PCI* prioritization criterion, e.g. in (11) $V_{1,2} = e$ since operation R_1 depends on the completion time of R_2, because $PCI_2 > PCI_1$, which yields $V_{2,1} = \varepsilon$ in (12), and so forth. Hence, from (11-17), the following (max,+)-linear system is obtained:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} = \begin{pmatrix} \cdot & p_2 & \cdot & \cdot & \cdot & p_6 & p_7 \\ \cdot & \cdot & \cdot & \cdot & \cdot & p_6 & p_7 \\ \cdot & \cdot & \cdot & \cdot & \cdot & p_6 & p_7 \\ \cdot & \cdot & p_3 & \cdot & \cdot & \cdot & p_7 \\ \cdot & p_2 & p_3 & p_4 & \cdot & p_6 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & p_6 & \cdot \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} \oplus \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \end{pmatrix}$$

Since the system is (max,+)-linear, the model's structure is quite simple and intuitive. This linearity property can be explored eventually as it is done for classic linear systems in conventional algebra. This is however not the focus of this work.

For simplicity, let the arrival dates be: $u = [e,e,e,e,e,e,e]^T$, i.e. all tankers arrive at $t_0 = 0$. The obtained schedule is as shown in Fig. 4. Through this schedule, resource allocation is done according to exploitation conditions of the network and the underlying costs that could be generated due to device malfunctioning. Notice that in other scenarios other than the one shown in Fig. 4 some operations could be executed simultaneously, therefore reducing the makespan. However, knowing that through the *PCI* vector operations are already ranked, this forced simultaneous execution would actually reduce device reliability and increase failure risk for following tasks with higher priority. For example, R_4 could be executed from $t_0 = 0$ simultaneously with R_6 but this would decrease resource reliability for R_7 which would be executed later and has higher priority.

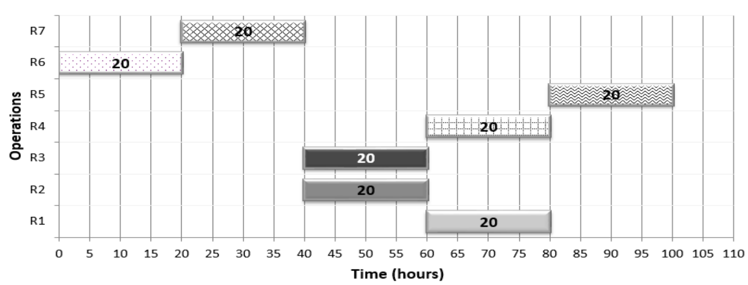

Fig. 4 Prioritization of Oil Operations

7 Concluding Remarks

The proposed approach exploits a (max,+) optimization model in order to schedule operations in a way that conflict resolution is managed through prioritization of operations according to a cost-reliability relation. The proposal approaches the case where device reliability can vary in a short-term, therefore affecting operative capacity with consequent costs related to penalties due to late service. Some other approaches have been explored focused on minimizing the TCP (see [18]). The approach proposed in this paper does not aim at replacing these previous results but is rather complementary, enriching the information that can be provided to supervision operators in order to improve decision making in a given operational situation.

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