

Representation of a reactive system with different models

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Abstract-- In this paper, we propose an approach which considers different models of a process (multi-model approach) based on the supervision theory of Ramadge and Wonham (RW) [1] [2]. Our contribution enables us to take into account various models which represent different operating modes of the process. In this approach only modes that ensure the same operating mode are actives while the others must be put into their respective inactive state. The problem of commutation between all designed models is formalised by a proposed framework which allows to determine each model and the commutation conditions.

Keywords: modelling, reactive systems, operating modes, discrete event systems.

I. INTRODUCTION

In the supervisory control theory of Ramadge and Wonham [1] [2] [3] some control theory problems, such as synthesis of controlled dynamic invariant system by feedback, and concepts such as controllability and non blocking have been investigated. However, in this theory the plant is often a product of a number of simple components. Thus its state size increases exponentially with the number of components and synthesising a controller becomes laborious.

A standard way to handle state explosion is by decentralised control. This approach consists of decomposing a system to be controlled G into subsystems G_i [5] [6] [7] for which local supervisors are fairly easy to obtain. Furthermore, reactive systems are subject to failures. This type of systems must be flexible in order to behave under controlled risks and allowing continuity of service which represents the prime aim of this paper. Flexibility is expressed by different operating modes of the system. In this paper a decentralised approach is used to model each operating mode and strategy commutation from an operating mode to another one. In our case only one operating mode is active at the same time. A framework is proposed to ensure the commutation.

II. MODELLING OF A MULTI-MODEL REACTIVE SYSTEM

Guaranteed functioning under failure causing downgraded production, yet allowing continuity of service, represents the prime aim of this section.

Reactive systems are subject to failures. This type of system must be flexible in order to behave under controlled risks. This flexibility is expressed by different operating modes of the system. In this section we are interested in the modelling of these operating modes by applying a multi-model concept which consists of designing a model process for each operating mode. The problem of commutation between all designed models is formalised by a proposed framework.

To introduce this formal framework, we consider a simple example and we will be limited to two models of the system. In figure 1.b two different models of a global system (Unit production) are represented. This system is composed of three machines as shown in fig. 1.

Initially the buffer is empty and M_3 is carrying out another task outside the unit but which intervenes when M_1 breaks down. With the event b_1 , M_1 takes a workpiece from an infinite bin and enters q_1 state but deposits it in the buffer B after completing its work. M_2 operates similarly, but takes its workpiece from B and deposits it when finished in an infinite output bin.

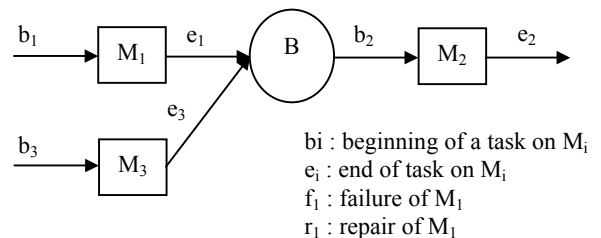


Fig. 1. Scheme of Production unit

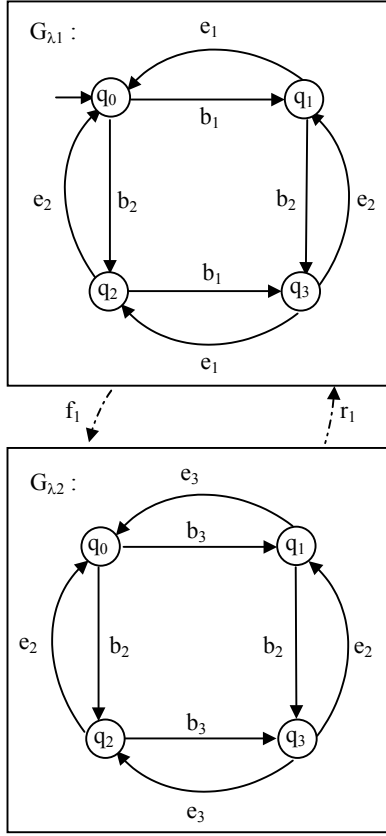


Fig. 2. Two possible models of the production unit

Now the aim is to determine each model and the commutation conditions. For this, we define Λ as a set containing indices of all models composing the global system with $\text{card}(\Lambda) = n < \infty$.

$\text{Card}(\Lambda)$ represents the number of models to be designed. In our case $\Lambda := \{\lambda_1, \lambda_2\}$ so $\text{card}(\Lambda) = 2$.

Let $\lambda_i \in \Lambda$, we define G_{λ_i} as an uncontrollable DES which is taken to be an automaton of the model λ_i

$$G_{\lambda_i} = (Q_{\lambda_i}, \delta_{\lambda_i}, \Sigma_{\lambda_i}, q_{0,\lambda_i}, Q_{m,\lambda_i})^1.$$

We suppose that $\Sigma_{\lambda_i} \cap \Sigma_{\lambda_j} \neq \emptyset$ ($i \neq j$) and initially the process model is G_{λ_1} . Let $\Sigma' = \cup_{ij} \{\alpha_{\lambda_i, \lambda_j}\}$ with $\alpha_{\lambda_i, \lambda_j}$ represents the commutation events from G_{λ_i} to G_{λ_j} . In our example $\Sigma' = \{f_1, r_1\}$. At the occurrence of $\alpha_{\lambda_i, \lambda_j}$ the process model becomes G_{λ_j} . However, in this case, we must determine the reception state of G_{λ_j} after the commutation. To do this, we extend G_{λ_i} and G_{λ_j} by adding respectively an inactive state q_{in, λ_i} to the state set of the model G_{λ_i} and an inactive state q_{in, λ_j} to G_{λ_j} state set. At the occurrence of $\alpha_{\lambda_i, \lambda_j}$, G_{λ_i} will be lead to q_{in, λ_i} and G_{λ_j} will be activated from q_{in, λ_j} . However, the problem is to determine the arrival state of G_{λ_j} (respectively G_{λ_i}) at the occurrence of $\alpha_{\lambda_i, \lambda_j}$ (respectively $\alpha_{\lambda_j, \lambda_i}$).

¹ Q_{λ_i} : Set of states, Σ_{λ_i} : the set of alphabet, δ_{λ_i} : the function transition, q_{0,λ_i} : the initial state and Q_{m,λ_i} : the Set of marked states.

Let $G_{\lambda_i, \text{ext}} = (Q_{\lambda_i, \text{ext}}, \delta_{\lambda_i, \text{ext}}, \Sigma_{\lambda_i, \text{ext}}, q_{0,\lambda_i, \text{ext}}, Q_{m,\lambda_i, \text{ext}})$ with :

- $Q_{\lambda_i, \text{ext}} = Q_{\lambda_i} \cup \{q_{in, \lambda_i}\}$
 - $\Sigma_{\lambda_i, \text{ext}} = \Sigma_{\lambda_i} \cup \Sigma'$
 - $q_{0,\lambda_i, \text{ext}} = q_{0,\lambda_i}$
 - $Q_{m,\lambda_i, \text{ext}} = Q_{m,\lambda_i}$
 - $\delta_{\lambda_i, \text{ext}}$ is defined as follows :
 1. $\forall q \in Q_{\lambda_i}$ and $\forall \sigma \in \Sigma_{\lambda_i}$, if $(\delta_{\lambda_i}(q, \sigma) \neq \emptyset)$, then $\delta_{\lambda_i, \text{ext}}(q, \sigma) = \delta_{\lambda_i}(q, \sigma)$.
 2. $\forall q \in Q_{\lambda_i}$ from which $\alpha_{\lambda_i, \lambda_j}$ can occur, then $\delta_{\lambda_i, \text{ext}}(q, \alpha_{\lambda_i, \lambda_j}) = q_{in, \lambda_i}$.
- $\delta_{\lambda_i, \text{ext}}(q_{in, \lambda_i}, \alpha_{\lambda_j, \lambda_i})$ will be defined later.

Now let us define $G_{\lambda_j, \text{ext}}$ to be the extended model of G_{λ_j} .

$G_{\lambda_j, \text{ext}} = (Q_{\lambda_j, \text{ext}}, \delta_{\lambda_j, \text{ext}}, \Sigma_{\lambda_j, \text{ext}}, q_{0,\lambda_j, \text{ext}}, Q_{m,\lambda_j, \text{ext}})$ with :

- $Q_{\lambda_j, \text{ext}} = Q_{\lambda_j} \cup \{q_{in, \lambda_j}\}$
- $\Sigma_{\lambda_j, \text{ext}} = \Sigma_{\lambda_j} \cup \Sigma'$
- $q_{0,\lambda_j, \text{ext}} = q_{in, \lambda_j}$
- $Q_{m,\lambda_j, \text{ext}} = Q_{m,\lambda_j}$
- $\delta_{\lambda_j, \text{ext}}$ is defined as follows:
 1. $\forall q \in Q_{\lambda_j}$ and $\forall \sigma \in \Sigma_{\lambda_j}$, if $(\delta_{\lambda_j}(q, \sigma) \neq \emptyset)$ then $\delta_{\lambda_j, \text{ext}}(q, \sigma) = \delta_{\lambda_j}(q, \sigma)$.
 2. $\forall q \in Q_{\lambda_j}$ from which $\alpha_{\lambda_j, \lambda_i}$ can occur, then $\delta_{\lambda_j, \text{ext}}(q, \alpha_{\lambda_j, \lambda_i}) = q_{in, \lambda_j}$.

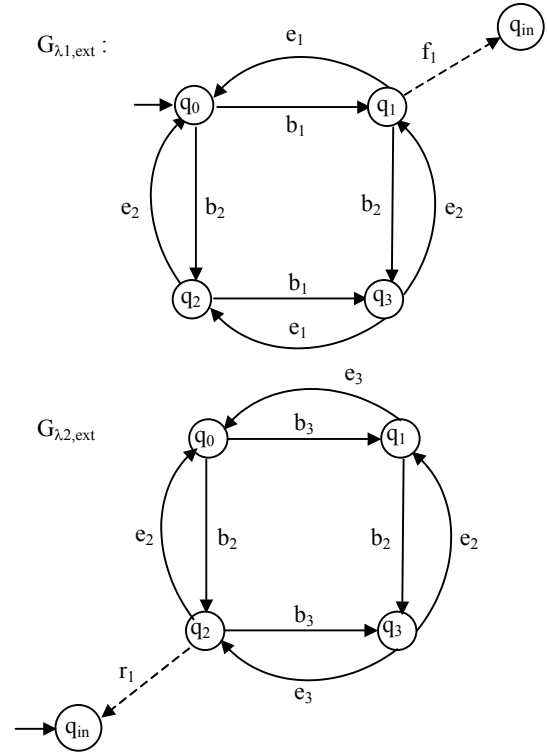


Fig. 3. Extended models of G_{λ_1} and G_{λ_2}

The objective now is to define $\delta_{\lambda_j, \text{ext}}(q_{in, \lambda_j}, \alpha_{\lambda_i, \lambda_j})$.

² $\delta_{\lambda_i}(q, \sigma) \neq \emptyset$ means that $\delta_{\lambda_i}(q, \sigma)$ is defined

Note that initially $G_{\lambda_j, \text{ext}}$ is in inactive state q_{in, λ_j} . At the occurrence of an event $\alpha_{\lambda_i, \lambda_j}$, $G_{\lambda_j, \text{ext}}$ must leave q_{in, λ_j} in order to reach a state $q \in Q_{\lambda_j}$. As shown in fig. 3, at the occurrence of event f_1 , $G_{\lambda_2, \text{ext}}$ which is in q_{in} , will be lead to q_0 , q_1 , q_2 or q_3 . So $G_{\lambda_2, \text{ext}}$ becomes nondeterministic. In order to avoid this nondeterministic situation, we propose the following procedure :

Let $R(G_{\lambda_j}, q_{\text{in}, \lambda_j}, \alpha_{\lambda_i, \lambda_j})$ be the set of reachable states from q_{in, λ_j} by the occurrence of $\alpha_{\lambda_i, \lambda_j}$. To determine this set, we introduce $\pi_{\lambda_i, \lambda_j} : L_{\alpha_{\lambda_i, \lambda_j}}(G_{\lambda_i}, q_{0, \lambda_i})^3 \rightarrow (\Sigma_{\lambda_i} \cap \Sigma_{\lambda_j})^*$ with :

$$\pi_{\lambda_i, \lambda_j}(\varepsilon) = \varepsilon \text{ and}$$

$$\pi_{\lambda_i, \lambda_j}(s\sigma) = \begin{cases} \pi_{\lambda_i, \lambda_j}(s)\sigma & \text{if } \sigma \in (\Sigma_{\lambda_i} \cap \Sigma_{\lambda_j}). \\ \pi_{\lambda_i, \lambda_j}(s) & \text{otherwise.} \end{cases}$$

That is, $\pi_{\lambda_i, \lambda_j}$ is a projection whose effect on a string $s \in (\Sigma_{\lambda_i})^*$ is to erase the elements σ of s that do not belong to $(\Sigma_{\lambda_i} \cap \Sigma_{\lambda_j})$. $\pi_{\lambda_i, \lambda_j}(s\sigma)$ allows the identification from G_{λ_j} of the output states of the intersection elements of G_{λ_i} when $\alpha_{\lambda_i, \lambda_j}$ occurs. We achieve the projection definition by defining $(\pi_{\lambda_i, \lambda_j}(s\sigma))_f$ as the last event of string $s\sigma$ over $\pi_{\lambda_i, \lambda_j}$.

For example, in the fig. 3, we can determine $\pi_{\lambda_1, \lambda_2}(b_1) = \varepsilon$, $\pi_{\lambda_1, \lambda_2}(b_1b_2) = b_2$ and $\pi_{\lambda_1, \lambda_2}(b_2e_2b_1) = b_2e_2$ then $(\pi_{\lambda_1, \lambda_2}(s\sigma))_f = (\pi_{\lambda_1, \lambda_2}(b_2e_2b_1))_f = e_2$.

Now from the definition of $\pi_{\lambda_i, \lambda_j}$ two cases are possible : $(\pi_{\lambda_i, \lambda_j}(s\sigma))_f = \varepsilon$ (case1) or $(\pi_{\lambda_i, \lambda_j}(s\sigma))_f \neq \varepsilon$ (case 2).

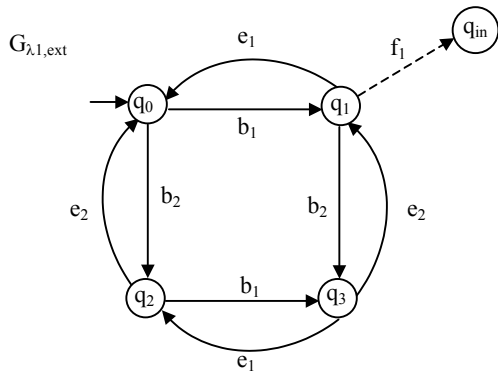
Case 1 :

$(\pi_{\lambda_i, \lambda_j}(s\sigma))_f = \varepsilon$ means that no event of $(\Sigma_{\lambda_i} \cap \Sigma_{\lambda_j})$ has occurred i.e. no intersection element works.

For example, from fig. 4 we assume that only M_1 is working and G_{λ_1} is in q_1 (because of the possible generation of f_1 after the generation of b_1).

Since no event of Σ_{λ_2} has occurred. So at the occurrence of f_1 , G_{λ_2} will be lead to the initial state q_0 of G_{λ_2} . Thus $R(G_{\lambda_2}, q_{\text{in}}, f_1) = q_0$ and $\delta_{\lambda_2, \text{ext}}(q_{\text{in}}, f_1) = q_0$.

Generally if $(\pi_{\lambda_i, \lambda_j}(s\sigma))_f = \varepsilon$, then $R(G_{\lambda_j}, q_{\text{in}, \lambda_j}, \alpha_{\lambda_i, \lambda_j}) = q_{0, \lambda_j}$ and so $\delta_{\lambda_j, \text{ext}}(q_{\text{in}, \lambda_j}, \alpha_{\lambda_i, \lambda_j}) = q_{0, \lambda_j}$.



³ $L_{\alpha_{\lambda_i, \lambda_j}}(G_{\lambda_i}, q_{0, \lambda_i}) := \{s \in L(G_{\lambda_i}) / \text{post}(s) = \alpha_{\lambda_i, \lambda_j}\}$ with $\text{post}(s)$ represents the next event to be occurred after the generation of s .

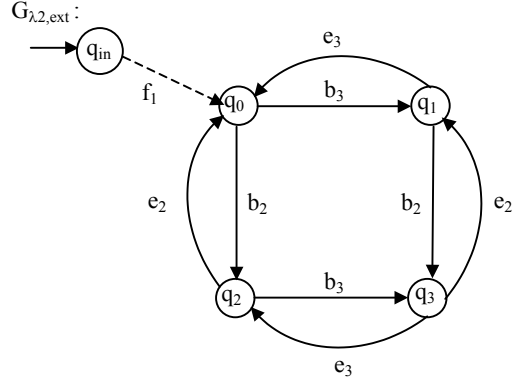


Fig. 4. Extended models of G_{λ_1} and G_{λ_2} for case 1

Case 2 :

Now suppose that $(\pi_{\lambda_i, \lambda_j}(s\sigma))_f \neq \varepsilon$ i.e. at least one intersection element is working.

For example, Suppose that in G_{λ_1} of fig. 4, b_2 has occurred. Then $(\pi_{\lambda_1, \lambda_2}(b_2))_f = b_2$. So from q_{in} , G_{λ_2} can be lead to q_2 or q_3 thus presenting a nondeterministic. To avoid this situation we introduce the following lemma whose proof is not provided here because of the space limitation.

Lemma 1 :

$\delta_{\lambda_j, \text{ext}}(q_{\text{in}, \lambda_j}, \alpha_{\lambda_i, \lambda_j})$ is an unique state which is given by $\delta_{\lambda_j, \text{ext}}(q_{\text{in}, \lambda_j}, \alpha_{\lambda_i, \lambda_j}) = \delta_{\lambda_j}(q_{0, \lambda_j}, \pi_{\lambda_i, \lambda_j}(s\sigma))$.

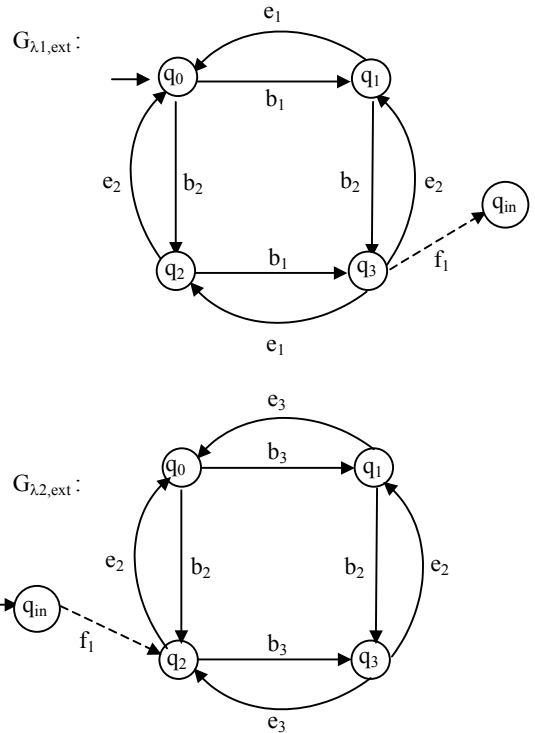


Fig. 5 Extended models of G_{λ_1} and G_{λ_2} for case 2

From fig. 5 :

$\delta_{\lambda,2,ext}(q_{in}, f_1) = \delta_{\lambda,2}(q_0, \pi_{\lambda,1,\lambda,2}(b_1b_2)) = \delta_{\lambda,2}(q_0, b_2) = q_2$. Thus if $(\pi_{\lambda,i,\lambda,j}(s\sigma))_f \neq \varepsilon$ then $\delta_{\lambda,j,ext}(q_{in,\lambda,i}, \alpha_{\lambda,j,\lambda,i}) = \delta_{\lambda,j,ext}(q_0,\lambda,i, \pi_{\lambda,i,\lambda,j}(s\sigma))$.

Now suppose that $G_{\lambda,i}$ is inactive i.e. $G_{\lambda,i}$ is in $q_{in,\lambda,i}$ and $G_{\lambda,j}$ is active i.e. $G_{\lambda,j}$ is in a state $q_{\lambda,j} \in Q_{\lambda,j}$. If the event $\alpha_{\lambda,j,\lambda,i}$ occurs, $G_{\lambda,j}$ will be inactive but $G_{\lambda,i}$ will leave $q_{in,\lambda,i}$ to a state $q_{\lambda,i} \in Q_{\lambda,i}$. We must then as previously define $\delta_{\lambda,i,ext}(q_{in,\lambda,i}, \alpha_{\lambda,j,\lambda,i})$. To do this, we introduce $\pi_{\lambda,j,\lambda,i}$ which is defined as follows:

$$\pi_{\lambda,j,\lambda,i}: L_{\alpha_{\lambda,j,\lambda,i}}(G_{\lambda,j}, q_{in,\lambda,i}) \rightarrow (\Sigma_{\lambda,i} \cap \Sigma_{\lambda,j})^*$$

$\pi_{\lambda,j,\lambda,i}(\varepsilon) = \varepsilon$ and

$$\pi_{\lambda,j,\lambda,i}(s\sigma) = \begin{cases} \pi_{\lambda,j,\lambda,i}(s)\sigma & \text{if } \sigma \in (\Sigma_{\lambda,i} \cap \Sigma_{\lambda,j}). \\ \pi_{\lambda,j,\lambda,i}(s) & \text{otherwise.} \end{cases}$$

For example, from fig. 6 $\pi_{\lambda,2,\lambda,1}(b_3) = \pi_{\lambda,2,\lambda,1}(b_3e_3) = \varepsilon$, but $\pi_{\lambda,2,\lambda,1}(b_3b_2) = b_2$ and $\pi_{\lambda,2,\lambda,1}(b_3b_2e_2) = b_2e_2$.

From the definition of $\pi_{\lambda,j,\lambda,i}$ two cases, are possible : $(\pi_{\lambda,j,\lambda,i}(s\sigma))_f = \varepsilon$ (case 3) or $(\pi_{\lambda,j,\lambda,i}(s\sigma))_f \neq \varepsilon$ (case 4).

Case 3 :

$(\pi_{\lambda,j,\lambda,i}(s\sigma))_f = \varepsilon$ means that no event of $\Sigma_{\lambda,i} \cap \Sigma_{\lambda,j}$ has occurred and $G_{\lambda,i}$ must be lead to a state where the intersection elements of $G_{\lambda,i}$ and $G_{\lambda,j}$ are respectively in their initial states. From fig. 6, q_0 and q_1 are possible in $G_{\lambda,1,ext}$. The objective is to keep only one state by consulting $(\pi_{\lambda,i,\lambda,j}(s'\sigma'))_f$.

Case 3.a : if $(\pi_{\lambda,i,\lambda,j}(s'\sigma'))_f = \varepsilon$ then $R(G_{\lambda,j}, q_{in,\lambda,j}, \alpha_{\lambda,i,\lambda,j}) = q_{0,\lambda,j}$. So from $q_{0,\lambda,j}$ and $(\pi_{\lambda,i,\lambda,j}(s'\sigma'))_f = \varepsilon$ it can be seen that the intersection elements are in their initial state in $G_{\lambda,j}$. Thus when commuting from $G_{\lambda,j}$ to $G_{\lambda,i}$ we will lead $G_{\lambda,i}$ to a state where the intersection elements are in their initial state. This state is inevitably $q_{0,\lambda,i}$. Consequently $\delta_{\lambda,i,ext}(q_{in,\lambda,i}, \alpha_{\lambda,j,\lambda,i}) = q_{0,\lambda,i}$.

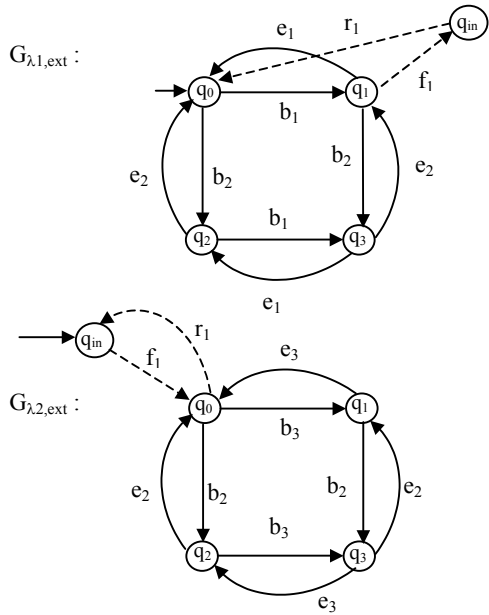


Fig. 6. Extended models of $G_{\lambda,1}$ and $G_{\lambda,2}$ for case 3.a

Case 3.b : if $(\pi_{\lambda,i,\lambda,j}(s'\sigma'))_f \neq \varepsilon$ then the intersection elements stay at $\delta_{\lambda,i}(q_{0,\lambda,i}, \pi_{\lambda,i,\lambda,j}(s'\sigma))$.

Thus $\delta_{\lambda,i,ext}(q_{in,\lambda,i}, \alpha_{\lambda,j,\lambda,i}) = \delta_{\lambda,i}(q_{0,\lambda,i}, \pi_{\lambda,i,\lambda,j}(s'\sigma))$.

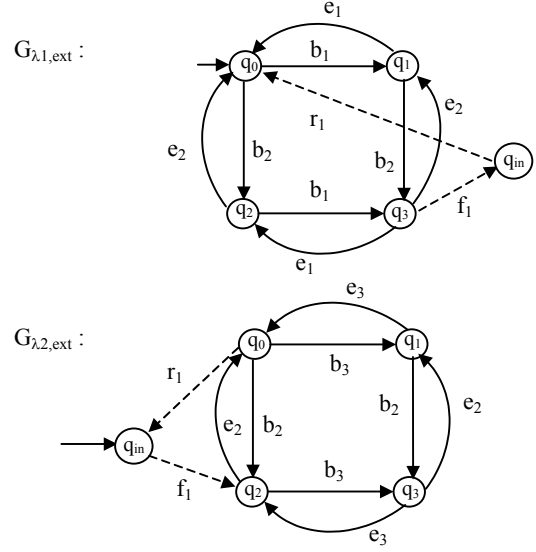


Fig. 7. Extended models of $G_{\lambda,1}$ and $G_{\lambda,2}$ for case 3.b

Case 4 :

Now suppose that $(\pi_{\lambda,j,\lambda,i}(s\sigma))_f \neq \varepsilon$. At this level two cases can be differentiated : $(\pi_{\lambda,i,\lambda,j}(s'\sigma))_f = \varepsilon$ or $(\pi_{\lambda,i,\lambda,j}(s'\sigma))_f \neq \varepsilon$.

Case 4.a : $(\pi_{\lambda,j,\lambda,i}(s\sigma))_f \neq \varepsilon$ and $(\pi_{\lambda,i,\lambda,j}(s'\sigma))_f = \varepsilon$. This means that before commutation, no event in $(\Sigma_{\lambda,i} \cap \Sigma_{\lambda,j})$ has occurred. As shown in case 1, if $(\pi_{\lambda,i,\lambda,j}(s'\sigma))_f = \varepsilon$ then $R(G_{\lambda,j}, q_{in,\lambda,j}, \alpha_{\lambda,i,\lambda,j}) = q_{0,\lambda,j}$. In the other hand $(\pi_{\lambda,j,\lambda,i}(s\sigma))_f \neq \varepsilon$ so the events in $(\Sigma_{\lambda,i} \cap \Sigma_{\lambda,j})$ have occurred from $q_{0,\lambda,j}$. Thus $R(G_{\lambda,i}, q_{in,\lambda,i}, \alpha_{\lambda,j,\lambda,i}) = \delta_{\lambda,i}(q_{0,\lambda,i}, \pi_{\lambda,j,\lambda,i}(s\sigma))$. Consequently $\delta_{\lambda,i,ext}(q_{in,\lambda,i}, \alpha_{\lambda,j,\lambda,i}) = \delta_{\lambda,i}(q_{0,\lambda,i}, \pi_{\lambda,j,\lambda,i}(s\sigma))$.

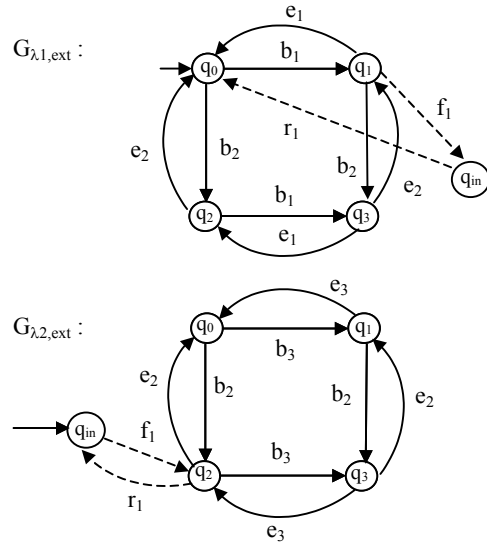


Fig. 8. Extended models of $G_{\lambda,1}$ and $G_{\lambda,2}$ for case 4.a

Case 4.b : $(\pi_{\lambda_j, \lambda_i}(s\sigma))_f \neq \varepsilon$ and $(\pi_{\lambda_i, \lambda_j}(s'\sigma'))_f \neq \varepsilon$.
 $(\pi_{\lambda_i, \lambda_j}(s'\sigma'))_f \neq \varepsilon$ lead to $R(G_{\lambda_j}, q_{in, \lambda_j}, \alpha_{\lambda_i, \lambda_j}) = \delta_{\lambda_j}(q_{0, \lambda_j}, \pi_{\lambda_i, \lambda_j}(s'\sigma'))$ (case 2).
 $(\pi_{\lambda_j, \lambda_i}(s\sigma))_f \neq \varepsilon$ means that events in $(\Sigma_{\lambda_i} \cap \Sigma_{\lambda_j})$ have occurred from q_{0, λ_j} i.e. $\delta_{\lambda_j}(q_{0, \lambda_j}, \pi_{\lambda_j, \lambda_i}(s\sigma))!$ in G_{λ_j} . We conclude that $\delta_{\lambda_i}(q_{in, \lambda_i}, \alpha_{\lambda_j, \lambda_i}) = \delta_{\lambda_i}(q_{0, \lambda_i}, \pi_{\lambda_i, \lambda_j}(s'\sigma) \pi_{\lambda_j, \lambda_i}(s\sigma))$ i.e.
 $R(G_{\lambda_i}, q_{in, \lambda_i}, \alpha_{\lambda_j, \lambda_i}) = \delta_{\lambda_i}(q_{0, \lambda_i}, \pi_{\lambda_i, \lambda_j}(s'\sigma) \pi_{\lambda_j, \lambda_i}(s\sigma))$.

From figure 4.b.2, we suppose that $\pi_{\lambda_1, \lambda_2}(s'\sigma) = b_2$ then $\delta_{\lambda_2}(q_0, b_2) = q_2$ in G_{λ_2} . Now we can introduce the lemma 2 which allows to determine the arrival state when commuting from G_{λ_j} to G_{λ_i} .

Lemma 2 :

$\delta_{\lambda_i, ext}(q_{in, \lambda_i}, \alpha_{\lambda_j, \lambda_i})$ is an unique state which is given by
 $\delta_{\lambda_i, ext}(q_{in, \lambda_i}, \alpha_{\lambda_j, \lambda_i}) = \delta_{\lambda_i}(q_{0, \lambda_i}, \pi_{\lambda_i, \lambda_j}(s'\sigma) \pi_{\lambda_j, \lambda_i}(s\sigma))$.

$(\pi_{\lambda_2, \lambda_1}(s\sigma))_f \neq \varepsilon$ for example $(\pi_{\lambda_2, \lambda_1}(b_3e_2))_f = e_2$ so machine M_2 has finished its task.

Thus $\delta_{\lambda_1, ext}(q_{in}, r_1) = \delta_{\lambda_1}(q_0, \pi_{\lambda_1, \lambda_2}(b_2) \pi_{\lambda_2, \lambda_1}(b_3e_2))$
 $= \delta_{\lambda_1}(q_0, b_2e_2) = q_0$ in G_{λ_1} .

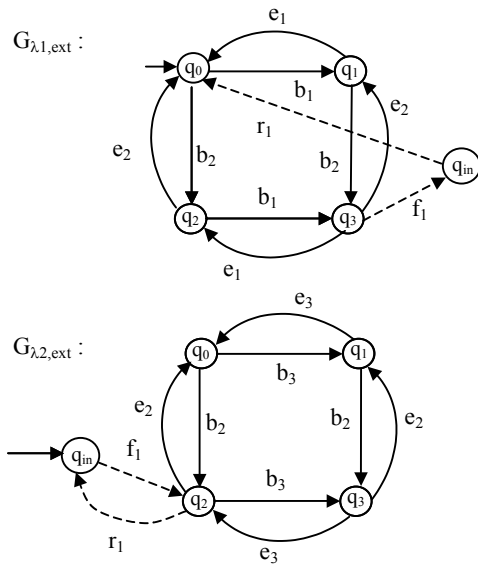


Fig. 9. Extended models of G_{λ_1} and G_{λ_2} for case 4.b

III. CONCLUSION

We conclude that the proposed method ensures commutation between different models of a global system reacting to exceptional situations such as a failure event occurrence. The major contribution of this paper considers reactive systems with different objectives. Each objective (or operating mode) is represented by a model of the process. Supposing that the different models evolve independently, the main problem is then to inactivate a model G_{λ_i} and to commute to a model G_{λ_j} which will be considered as the model of the process until the occurrence of an exceptional event. A formal framework

based on tracking events is proposed in order to ensure the commutation. This framework introduces a new projection definition.

Lemma 1 and 2 constitute the main result of this paper. They allow to determine the arrival state of a model after commutation.

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